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Behavior and Design of Angle Compression Members

Author

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Specializing in the structural analysis and design of unusual and complex structures, Lutz's work includes the structural analysis and design of skylight structures, reinforced concrete and steel tanks, tank covers, domes, space frames, as well as the seismic analysis of equipment and structures.

Lutz has written numerous specialized structural analysis and design computer programs for specific applications. In addition, he has written large scale analyses and reinforced concrete beam design programs, which are being sold internationally by ECOM Associates for minicomputer systems. He is also proficient in design and behavior of reinforced concrete structures and served as project structural engineer for the design of several large buildings.

Lutz has done research on cracking of concrete structures and bonding of reinforcement in concrete. The crack-control expression presently in the *Building Code Requirements for Reinforced Concrete* was taken from his doctoral work at Cornell University. He has written a number of papers for an ACI journal and AISC's *Engineering Journal.*

A fellow of ACI, Lutz is also a member of ASCE. He is particularly active in ACI, where he is a member of Committee 408—Bond and Anchorage, Committee 224—Cracking and Committee 439—Steel Reinforcement and has served as chairman of committees 439 and 408. Lutz is also presently chairman of the SSRC Task Group 26—Stability of Angle Struts and is a member of the AISC ad hoc committee on single-angle members.

Summary

The provisions of the proposed AISC Appendix F, on the design of single-angle members, of the AISC *Specification for the Design, Fabrication and Erection of Structural Steel for Buildings are* presented. The tensile and shear provisions are summarized. The flexural and compression provisions are presented in greater detail for both equal and unequal leg angles.

The compression integrity of single angles may be affected by something other than flexural buckling. Provisions for local buckling are presented as are requirements for evaluating torsional-flexural buckling. Information is given to evaluate when local and torsional-flexural buckling controls.

The flexural integrity of single angles bent about either the principal or geometric axes is discussed. Lateral-torsional provisions for both flexural orientations are given, as are the local buckling limits on flexural stress.

The design of single-angle, beam-column members is illustrated with several examples. The beam column examples will show how to apply the flexural and compression provisions for use in the combined stress interaction expressions.

BEHAVIOR AND DESIGN OF ANGLE COMPRESSION MEMBERS

INTRODUCTION

Design of single-angle struts has been covered to this point in the AISC Construction Manual with a one page summary. This summary provides a cautionary outline of how such compression loaded struts should be treated for this design.

More detailed provisions have probably not been put into code form for several reasons - perceived lack of importance and analysis complexity. Single angles have always been considered as secondary structural members. The loading of single angle struts is typically eccentric, producing flexure about a non-principal axis. Furthermore, analysis may be complicated by the possibility of torsional or lateral buckling of the angle.

Proper analysis and design of single angle struts could only be accomplished with some effort. Effort is required to find appropriate governing provisions as well as applying them correctly. Lack of a precise procedure for design in a single document has led to use of simplified procedures which may be conservative or unconservative.

Single angles are used as primary structural components and members. Their analysis and design should be done properly to assure that inadequate single angles do not result. Necessary for the proper design of single angles is an understanding of the behavior of single angle struts.

Single angle struts or compression members are beam-columns. That is, they are subjected to flexural loading (usually about both principal axes) in addition to axial compression. The flexural integrity of the single angle will be examined as well as the axial integrity. Furthermore for completeness, the tensile integrity of single angles will be presented. Much of the background presented comes from a draft of "Design Criteria for Single Angle Members"*.

SINGLE ANGLES IN TENSION

Consistent with the design of other tensile members, under allowable stress design procedures, the allowable tensile stress, F_t, is limited to $0.6F_y$ of the gross area. This limit is spelled out in Sect. 1.5.1.1 of the ASD version of the AISC Specification[1].

Single angles, which are being considered as tensile struts and don't have any flexure induced by transverse load, may be considered as axially loaded struts for design purposes. In this case, even though there is some bending induced by the nature of the end attachment, the design can be adequately covered using the effective area concept. Combined stress concepts need be employed only when the member is subjected to transverse loads in addition to the tension.

The end connections produce a condition which is limited to a stress $F_t = 0.5\ F_u$ on the effective net area, A_e. F_u is the ultimate stress. Three types of end connections are given:

a) For members connected by bolting, A_e shall be determined by the net area and effective net area concepts from Sects. 1.14-1 through 1.14.4[1].

* Draft prepared by AISC Ad Hoc Committee on Design Criteria for Single Angle Members. Information from Figs. 1, 2, 4-6 and all expressions except eq. 8 and 9 are taken from this document.

b) For members connected by welding along one leg of the angle with longitudinal and combination of longitudinal and transverse welds, $A_e = 0.85\ A_g$.

c) For members connected by only a transverse weld on one leg of the angle, A_e shall be the area of that leg. The weld must be adequate to be able to employ this area.

In each case, however, the tensile stress F_t should also be limited to 0.6 F_y of the gross area.

SINGLE ANGLES IN COMPRESSION

Under uniaxial compression, the allowable stress F_a follows the provisions of Appendix C of the AISC Specification[1]. These provisions outlined in Fig. 1, in addition to determining the effects of column flexural buckling, consider the effects of local buckling by means of the factor Q. For unequal leg angles, the full width of the longer leg is used in the evaluation of the parameter b/t.

The largest b/t for hot rolled shapes is 20 which leads to a Q of 0.80 for A36 steel. Values of b/t in excess of this can occur in angles fabricated from bent plate. In this case Q will likely be calculated from Eq. 3c in Fig. 1.

For thin or unequal leg angles, flexural-torsional buckling will begin to control as the column becomes short. This situation can be determined by evaluating an equivalent slenderness ratio in place of KL/r.

$$(KL/r)_{equiv.} = \pi \sqrt{E/F_e} \qquad (4)$$

where F_e is the elastic buckling strength in the flexural torsional mode. This F_e can be determined from the provisions given in Fig. 2. These provisions are based on those of Appendix E of the LRFD Specification[2] with the angle's warping resistance being conservatively neglected.

For equal leg angles, flexural-torsional buckling will control if

$$(KL/r)_{max} < 5.4\ (b/t)/Q \qquad (7)$$

If b/t = 16 for an equal leg A36 angle, Q will be 0.911 and 5.4(b/t)/Q = 95. For unequal leg angles, flexural buckling will always be accompanied by some torsion. However, for larger slenderness ratios, the flexural-torsional buckling strength is approximately equal to the minimum flexural buckling stress.

Often the most difficult aspect of evaluation of column capacity for single angles is the determination of the effective slenderness ratio. The effective length factor can often be evaluated or estimated about the x and y axes of the angle. However, these are not the principal axes for the angle, so the determination of the governing slenderness ratio is not easy.

The most common situation has the ends of the angles attached with one leg to a chord stem or gusset plate. This connection generally produces a relatively rigid rotational restraint in the plane of the attached leg. The perpendicular leg usually has a small restraint due to the flexibility of the stem or gusset about the chord's axis. Due to the difference in effective lengths about the two geometric axes, the radius of gyration r_z should no longer represent the critical value.

The allowable compressive stress on the gross area of axially compressed single angle members shall be:

when $KL/r < C'_c$

$$F_a = \frac{Q\left[1 - \dfrac{(KL/r)^2}{2C'^2_c}\right]F_y}{5/3 + 3/8\left[\dfrac{KL/r}{C'_c}\right] - \dfrac{(KL/r)^3}{8C'^3_c}} \qquad (1)$$

when $KL/r \geq C'_c$

$$F_a = \frac{12\pi^2 E}{23(KL/r)^2} \qquad (2)$$

where

KL/r = largest effective slenderness ratio of any unbraced length as defined in Sect. 1.8[1] or equivalent slenderness ratio $\pi\sqrt{E/F_e}$ per Figure 2.

$$C'_c = \sqrt{\frac{2\pi^2 E}{QF_y}}$$

The reduction factor Q shall be:

when $b/t \leq 76/\sqrt{F_y}$

$$Q = 1 \qquad (3a)$$

when $76/\sqrt{F_y} < b/t \leq 155/\sqrt{F_y}$

$$Q = 1.340 - 0.00447\,(b/t)\sqrt{F_y} \qquad (3b)$$

when $b/t > 155/\sqrt{F_y}$

$$Q = 15{,}500/[F_y(b/t)^2] \qquad (3c)$$

where

b = full width of the longest angle leg

t = thickness of angle

Figure 1 - Allowable Compressive Stress for Single Angles

Equivalent slenderness ratio is $\pi\sqrt{E/F_e}$ (4)

where F_e, the elastic flexural-torsional buckling stress from Appendix E[2] with no warping resistance is:

1) For equal leg angles with w as the axis of symmetry:

$$F_e = \frac{F_{ew} + F_{ej}}{2H}\left[1 - \sqrt{1 - \frac{4 F_{ew} F_{ej} H}{\left(F_{ew} + F_{ej}\right)^2}}\right] \tag{5}$$

2) For unequal leg angles, F_e is the lowest root of the cubic equation:

$$(F_e - F_{ez})(F_e - F_{ew})(F_e - F_{ej}) - F_e^2 (F_e - F_{ew})(z_o/\bar{r}_o)^2 - F_e^2 (F_e - F_{ez})(w_o/\bar{r}_o)^2 = 0 \tag{6}$$

where

E = modulus of elasticity, ksi

G = shear modulus, ksi

J = torsional constant = $2bt^3/3 = t^2A/3$, in.^{4}

I_z, I_w = moment of inertia about principal axes, in.^{4}

z_o, w_o = coordinates of the shear center with respect to the centroid, in.

$\bar{r}_o^2 = z_o^2 + w_o^2 + (I_z + I_w)/A$, in.^{2} ; $I_z + I_w = I_x + I_y$

$H = 1 - (z_o^2 + w_o^2)/\bar{r}_o^2 = (I_z + I_w)/A\bar{r}_o^2$

$F_{ez} = \dfrac{\pi^2 E}{(K_z L/r_z)^2}$, ksi

$F_{ew} = \dfrac{\pi^2 E}{(K_w L/r_w)^2}$, ksi

$F_{ej} = \dfrac{GJ}{A\bar{r}_o^2} = \dfrac{Gt^2}{3\bar{r}_o^2}$, ksi

Figure 2 - Equivalent Slenderness from Elastic Flexural-Torsional Buckling

A suggested procedure for evaluating an effective moment of inertia or an effective radius of gyration for this situation is illustrated in Fig. 3. The method for evaluating the minimum moment of inertia for a cross-section (illustrated by its graphical solution) is modified by dividing the I_y and I_x values by the square of their respective effective length factors. This can be rewritten as r_{eff}, an effective radius of gyration, as indicated in Fig. 3.

This procedure simplifies to the correct values when $I_{xy} = 0$ where x and y are the principal axes and for the case where $K_y = K_x = 1$. This procedure appears to generally conform with angle behavior (both observed and anticipated) and is more conservative than some procedures suggested. This will be covered in more detail in Example Problem 3.

SINGLE ANGLES IN FLEXURE

Single angles in flexure will have their maximum stress at the tip of one of the legs. In some cases, such as equal leg angles bent about their z-axis, the stress at the angle's corner may reach the same level as that at the tips. However, it is safe to calculate only the tip stress. It is suggested that this stress be calculated at the center or mid-thickness at the end of the leg to achieve the most representative critical stress for all flexural directions.

The limiting stress for both tension and compression can be taken as 0.66 F_y. This is satisfactory since the shape factor for angles is in excess of 1.5. In compression this flexural stress limit may be lowered by local buckling or by lateral-torsional instability.

FLEXURAL LOCAL BUCKLING LIMITS

The b/t value of the leg whose tip is in compression is used to control the stress. If the tip of the leg is in tension local buckling need not be checked.

The limiting stress drops from 0.66 F_y to 0.60 F_y at $b/t = 65/\sqrt{F_y}$ and remains at 0.60 F_y to $b/t = 76/\sqrt{F_y}$. The allowable bending stress becomes

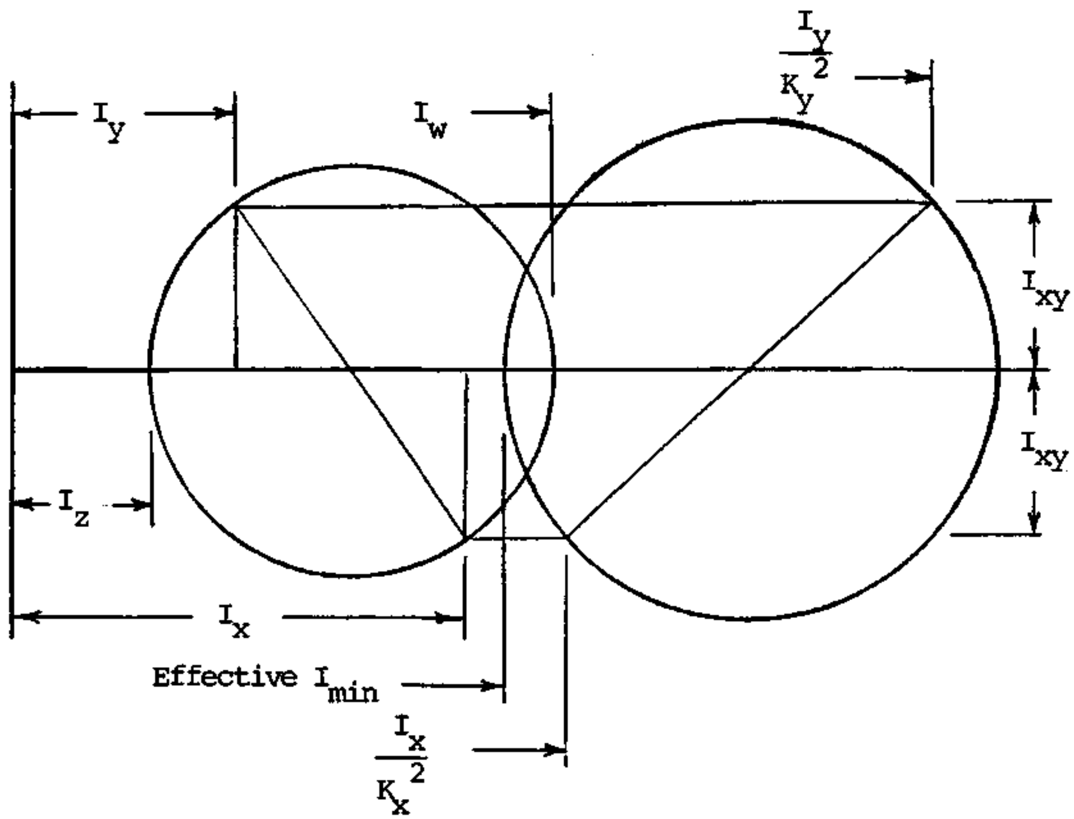
$$F_b = 0.60Q\,F_y \quad \text{for } b/t > 76/\sqrt{F_y} \tag{10}$$

where Q is as given in Fig. 1. This represents the same limit as used for unaxial compression. In flexure, since the compression drops to zero at or before reaching the corner of the angle, the limit is obviously conservative.

LATERAL-TORSIONAL BUCKLING LIMITS

An unbraced single angle in flexure can buckle in a lateral-torsional mode. This buckling can occur from bending of the angle about most any axis. Two parameters L/b and b/t influence the lateral-torsional integrity of single angles. In general, L/b reflects the lateral integrity and b/t reflects the torsional integrity.

The stress limit is based on the elastic lateral-torsional buckling stress, F_{ob} as shown in Fig. 4. These expressions are based on Australian research (3, 4, 5, 6). The elastic portion, when $F_{ob} < F_y$, has a variable factor of safety ranging from 2.22 to 1.82. The transition region limits the stress when F_{ob} is between F_y and approximately 3 F_y.



$$\text{Effective } I_{min} = \frac{I_y}{2K_y^2} + \frac{I_x}{2K_y^2} - \sqrt{\left[\frac{I_y}{2K_y^2} - \frac{I_x}{2K_x^2}\right]^2 + I_{xy}^2} \qquad (8)$$

r_{eff}, the corresponding effective radius of gyration:

$$r_{eff} = \sqrt{1/2\left[\left(\frac{r_y}{K_y}\right)^2 + \left(\frac{r_x}{K_x}\right)^2\right] - \sqrt{1/4\left[\left(\frac{r_y}{K_y}\right)^2 - \left(\frac{r_x}{K_x}\right)^2\right]^2 + \left(\frac{I_{xy}}{A}\right)^2}} \qquad (9)$$

For the case of equal leg, angles with $K_x = 1$ and $K_y = .65$ to reflect almost fixed conditions parallel to the attached leg,

$r_{eff} = 0.88 r_x$

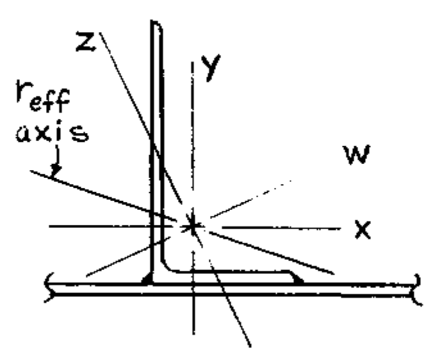


Figure 3 - Suggested Procedure for Effective Radius of Gyration

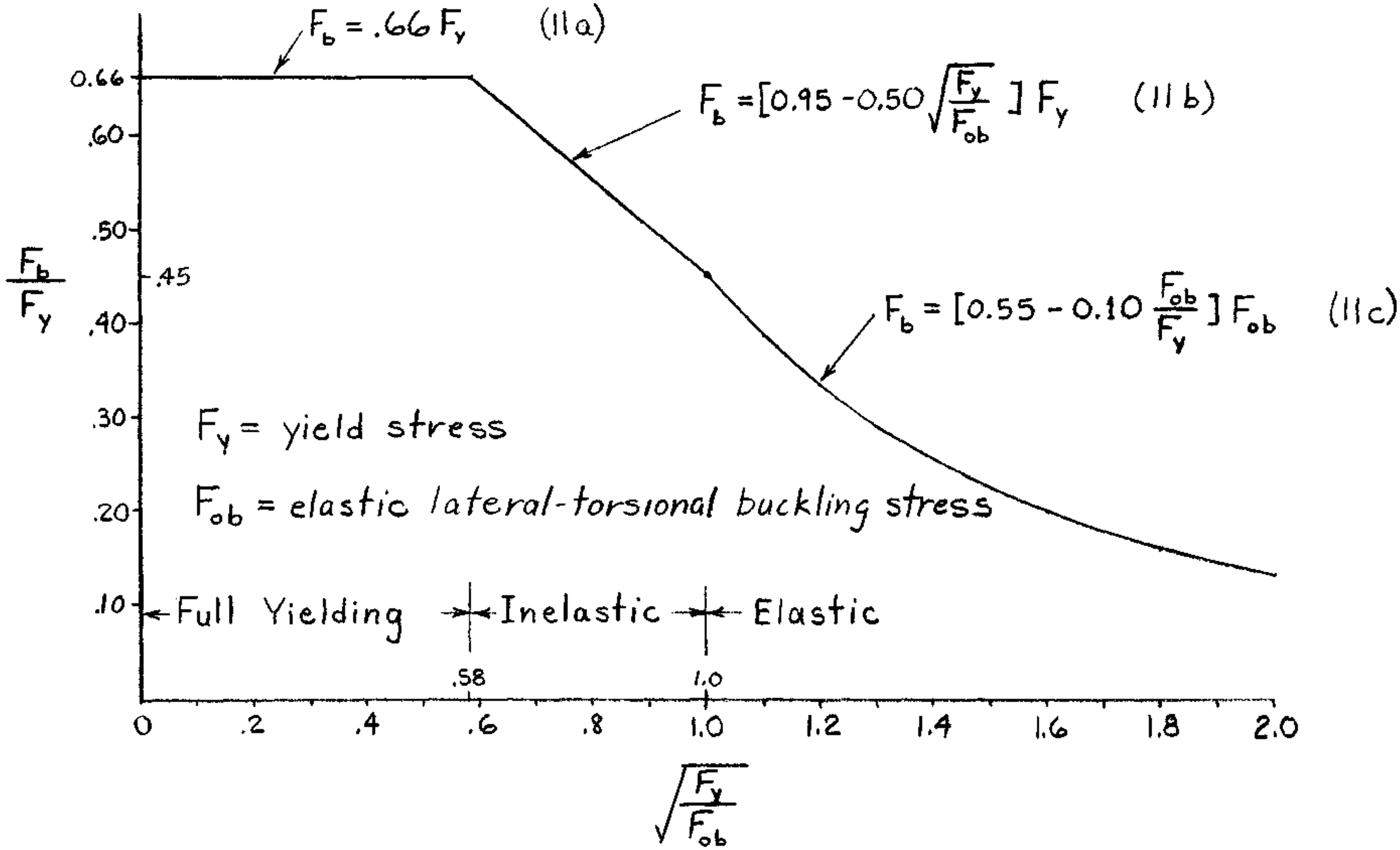


Figure 4 - Allowable Lateral-torsional Stress Limits

Principal Axis Buckling

The general expression for F_{ob} can be formulated into appropriate expressions for specific flexural conditions. First consider angles bent about their major principal axis. Angles bent about this axis are the most likely to exhibit lateral-torsional buckling tendencies.

The expression for F_{ob} for equal leg angles (designated F_{obw}) is

$$F_{obw} = \frac{\pi E t}{2\sqrt{2(1+\mu)}\, L}$$ which for steel becomes

$$F_{obw} = C_b \frac{28,250}{L/t} \qquad (12)$$

L is the unbraced length. C_b is introduced to consider the non-uniformity of the stress along the unbraced length L.

$C_b = 1.75 + 1.05(M_1/M_2) + 0.3(M_1/M_2)^2 \leq 1.5$, where M_1 is the smaller and M_2 is the larger end moment in the unbraced segment of the beam; (M_1/M_2) is positive when the moments cause reverse curvature and negative when bent in single curvature. C_b shall be taken as unity when the bending moment at any point within an unbraced length is larger than at both ends of its length.

Lateral-torsional buckling need not be checked if the parameter Lt/b^2 is less than $1.42C_b$; local buckling will control the value of F_b in this case.

The expression for F_{obw} for unequal leg angles bent about their major principal axis is considerably more complex than that for equal leg angles. This expression given in Fig. 5 contains the basic properties of the cross section I_z, r_z, S_w as well as the special section property β_w. For equal leg angles $\beta_w = 0$ and I_z, r_z, S_w can be expressed in terms of the length and thickness of a pair of plates representing the legs to permit the simplification shown above in Eq. 12.

The term β_w appears for unequal leg angles to reflect the presence of the shear center being either above or below the neutral axis. A lower F_{ow} results from use of a negative β_w when the shear center is below the neutral axis and the long leg is in compression.

Note that the stress limit F_{obw} and the stress f_{bw} are to be calculated using the section modulus for the tip or the leg in compression. If the smaller leg is in compression and the amount of axial compression is relatively small (or zero), the tensile stress should also be checked.

For minor axis bending, it is satisfactory to simply check local buckling to obtain appropriate stress limits. Local buckling should also be evaluated for the leg in compression under major axis bending.

BENDING OF ANGLES-GENERAL CASE

Angles are seldom bent about a single principal axis. Angles are most typically positioned such that bending is applied about one of the geometric (x and y) axes. This represents a biaxial bending situation about the principal axes.

The elastic lateral-torsional buckling stress about w-axis for unequal leg angles

$$F_{obw} = \frac{143{,}100 I_z}{L^2 S_w} C_b \left[\sqrt{\beta_w^2 + 0.052\,(Lt/r_z)^2} + \beta_w \right] \qquad (13)$$

S_w = section modulus to tip of compression leg, in.

I_z = minor principal axis moment of inertia, in.^{4}

r_z = radius of gyration for minor principal axis, in.

$\beta_w = \left[\frac{1}{I_w} \int_A z(w^2+z^2)\,dA \right] - 2z_o$, special section property for unequal leg angles, positive for short leg in compression and negative for long leg in compression, in. (See values below)

z_o = z distance from shear center to centroid, in.

I_w = major principal axis moment of inertia, in.^{4}

Angle Size (in.)	β_w (in.)
9 x 4	6.54
8 x 6	3.31
8 x 4	5.48
7 x 4	4.37
6 x 4	3.14
6 x 3.5	3.69
5 x 3.5	2.40
5 x 3	2.99
4 x 3.5	0.87
4 x 3	1.65
3.5 x 3	0.87
3.5 x 2.5	1.62
3 x 2.5	0.86
3 x 2	1.56
2.5 x 2	0.85

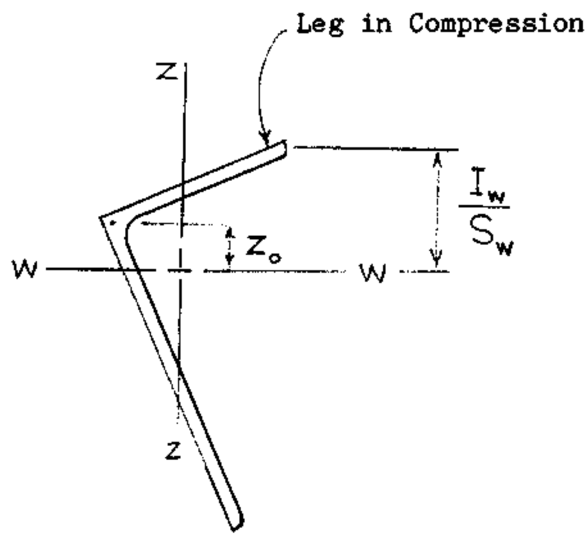


Figure 5 - F_{ob} for unequal leg angles bent about w-axis

For laterally unbraced angles, resolve the applied loading into components along the principal (z and w) axes of the angle. Separately evaluate the stresses and deflections of the angles for each of the principal axes. Evaluate the respective limits in flexure using the procedures outlined earlier.

The only exceptions to this general procedure are for the two situations outlined below. They are for laterally unrestrained equal leg angles and for laterally restrained angles bent about their geometric axes.

Geometric Axis Bending - Laterally Unrestrained

Two observations should be noted regarding the unbraced single angle bent by a load directed parallel to one of the angle's legs. First, the maximum stress will be larger than would have been calculated using the geometric axes section properties. Second, the angle deflects not only in the direction of load, but also perpendicular to the direction of load. The resultant deflection is larger than that obtained using the geometric axis moment of inertia with the total applied loading. It is unconservative to ignore the fact that a laterally unbraced single angle will tend to bend about the z-axis and deflect normal to that axis.

For an equal leg angles loaded parallel to one of its legs, it is relatively easy to evaluate the effects of the biaxial bending and simplify the analysis to that of an "uniaxial" one.

1. Calculate the maximum stress f_b at the tip of the leg (See Fig. 6) which is parallel to the direction of loading as

$$f_b = \frac{1.25 M_{max}}{S_x} \tag{14}$$

where M_{max} = maximum applied bending moment
S_x = Section Modulus for equal leg angle (Printed in section property tables)

2. Calculate the appropriate F_{ob} when the angle tips are in compression as

$$F_{obx} = \frac{85900}{(L/b)^2} C_b \left[\sqrt{1+0.78(Lt/b^2)^2} - 1 \right] \tag{15}$$

This need not be checked if $Lt/b^2 < 2.43$ (with $C_b = 1$), inasmuch as local buckling will control the allowable stress F_b.

3. The deflection of the angle is 1.82δ and is composed of two components 1.56δ and 0.94δ as shown in Fig. 6, where δ is evaluated using the total load with the geometric axis moment of inertia I_x.

For the case where the angle tips are in tension, the appropriate F_{obx} is that in item 2 with a plus 1 rather than a minus 1. For all practical cases $F_b = .66 F$ can be used as the stress limits. It takes an L/b ratio of 50 and b/t ratio of 24 before the F_{obx} becomes low enough such that the allowable stress drops below $0.66 F_y$ for A36 steel.

When axial compression of sufficient magnitude is imposed on the laterally unbraced angle shown in Fig. 6, the appropriate r_b must be evaluated for computing F_e'. Since the flexural deflection in the plane of bending is 1.82 times that calculated using I_x, the value of r_b should be $r_x/1.35$ since $1.35 = \sqrt{1.82}$.

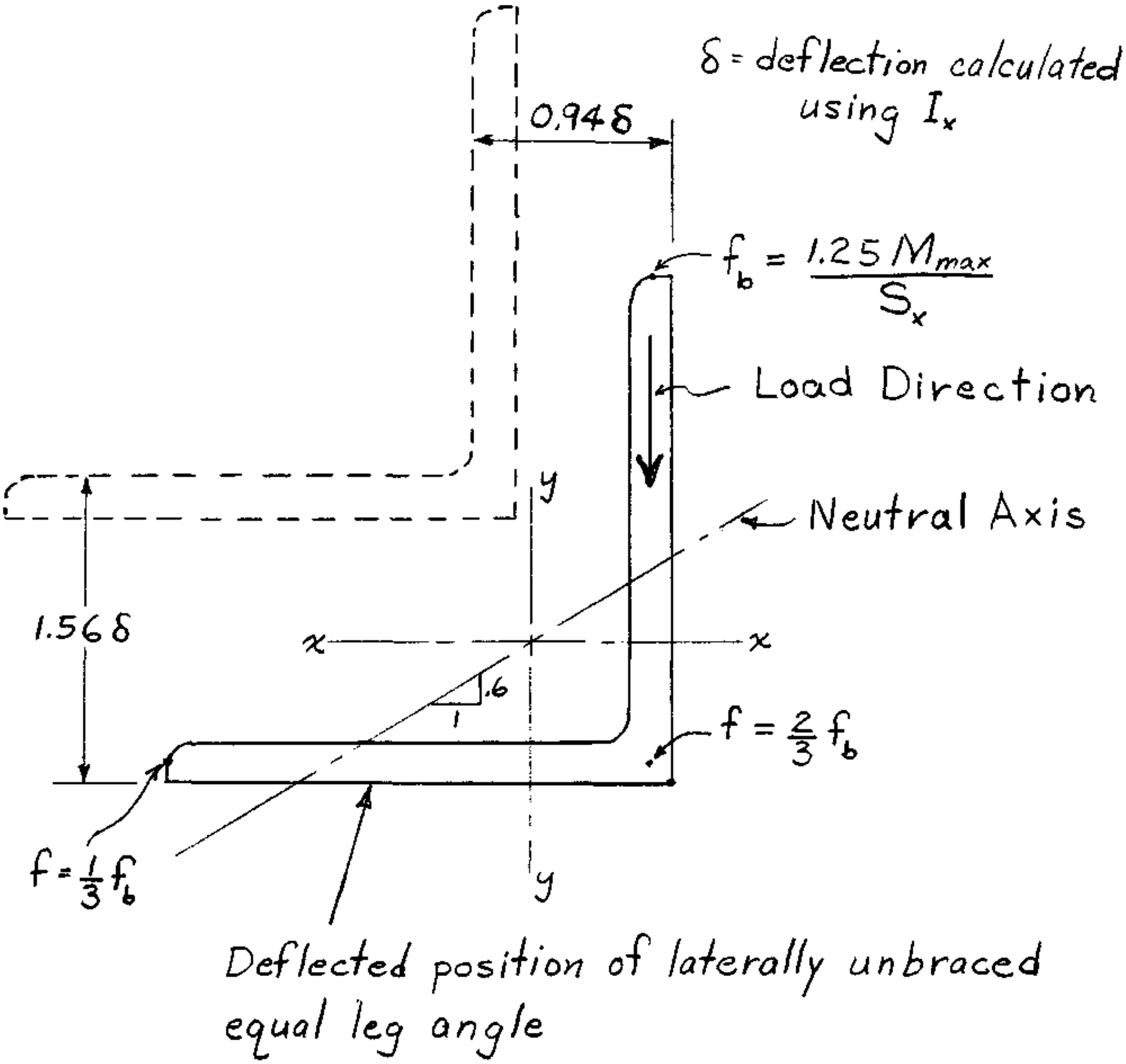


Figure 6 - Geometric Axis Bending of Equal Leg Angles

Geometric Axis Bending - Laterally Restrained

If the angle is fully restrained laterally along its entire span, the angle will deflect only in the direction of load. In this case, it is correct to calculate the maximum stress using S_x and the deflection using I_x. Only local buckling of the leg in compression need be checked. The radius of gyration r_b would be equal to r_x.

One should be aware that a lateral restraining force equal to about 60% of the applied load is required to maintain equal leg angles deflecting in the direction of load. This force can be more for unequal leg angles. If the brace is flexible such that the angle deflects laterally, the items noted in the previous paragraph are not valid.

For the relatively common case where the span is fully laterally-torsionally braced only at the point of maximum moment, the exact situation is not as easy to ascertain. It is deemed satisfactory in this case to calculate the maximum flexural stress as M_{max}/S_x. Thus, it would follow that maximum deflection should be based on I_x and that $r_b = r_x$.

Lateral-torsional buckling over the unbraced portion should also be examined using F_{obx} from Eq. 15. In this case C_b will likely be greater than 1. In most cases local buckling will likely be found to control the flexural stress limit rather than the lateral-torsional buckling.

SHEAR AND TORSION

The allowable shear stress is to be limited to $F_v = 0.4 F_y$. The flexural shear stress can conservatively be computed as $f_v = 1.5 V_b/bt$ where V_b is the shear force applied parallel to the angle leg of length b and thickness t. For the case of equal leg angles loaded along a geometric axis, the 1.5 factor may be replaced by 1.35.

The torsional shear stress can be computed as $f_v = M_t t/J$ which equals $3M_t/At$ where M_t is the torsional moment, A is the angle's cross sectional area and t the leg thickness. Research[5] has indicated that torsion need to considered only if the eccentricity of load exceeds half of the leg length.

COMBINED STRESSES

The combined stress expressions in Sect. 1.6.1 of the ASD specification[1] can be employed for angles with axial compression plus bending. The expressions are to be in terms of the principal axes for the angle.

$$\frac{f_a}{F_a} + \frac{C_{mw} f_{bw}}{\left[1 - \frac{f_a}{F'_{ew}}\right] F_{bw}} + \frac{C_{mz} f_{bz}}{\left[1 - \frac{f_a}{F'_{ez}}\right] F_{bz}} \leq 1 \qquad (16)$$

When $f_a/F_a \leq .15$ then

$$\frac{f_a}{F_a} + \frac{f_{bw}}{F_{bw}} + \frac{f_{bz}}{F_{bz}} \leq 1 \qquad (17)$$

In these equations f_{bw} and f_{bz} are maximum compression bending stresses in the member due to M_w and M_z respectively acting independently and may not occur at the same cross section.

Compression at a support location of the member should be checked using the following expression:

$$\frac{f_a}{0.60QF_y} + \frac{f_{bw}}{F_{bw}} + \frac{f_{bz}}{F_{bz}} \leq 1 \qquad (18)$$

In this case all stress values are evaluated at the same cross section.

For angles (as for tee beams) with a relatively small compression load, it is possible that the tension on a cross section could still control. With f_{bw} and f_{bz} as tensile stresses and $+f_a$ a compression stress

$$f_{bw} + f_{bz} - f_a \leq 0.66\ F_y \qquad (19)$$

For angles in axial tension and subjected to bending from transverse loading, check the stress at a cross section using the following expression:

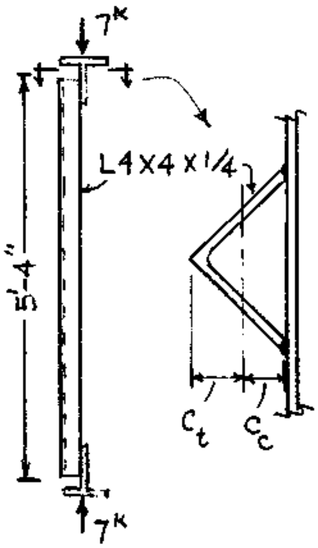
$$\frac{f_a}{0.60F_y} + \frac{f_{bw} + f_{bz}}{0.66F_y} \leq 1 \qquad (20)$$

EXAMPLE PROBLEMS

Three examples are presented to illustrate the use of many of the provisions that have been described in the previous sections.

Example Problem 1

Determine whether this angle strut can carry a 7^k axial load. This example includes consideration of local, flexural and flexural-torsional buckling.



$I_x = I_y = 3.04\ in.^4 \quad A = 1.94\ in.^2$

$x = y = 1.09"; \ r_z = 0.795$

$c_t = x\sqrt{2} = 1.54"$

$c_c = \frac{4 + .25/2}{\sqrt{2}} - c_t = 1.375"$

Consider eccentricity of load at 1.45"

$I_z = 1.94(.795)^2 = 1.226\ in.^4$

The steel is A36 material.

Consider unbraced length of 5'-4" = 64"

$KL/r = 64/.795 = 80.5$

$b/t = 4/.25 = 16$ $\quad Q = 1.34 - .00447(16)\ /36 = 0.911$

$C_c' = \sqrt{2\ \pi^2(29000)/(.911(36))} = 132.1$

$5.4(16)/.911 = 95 > 80.5$

Determine $(KL/r)_{equiv.} = \pi\sqrt{E/F_e}$ using Fig. 2

$w_o = \sqrt{2}(1.09-.25/2) = 1.365"$ $\quad I_z + I_w = I_x + I_y$

$\bar{r}_o^{\,2} = (1.365)^2 + 2(3.04)/1.94 = 4.996$

$H = 1 - (1.365)^2/4.996 = 0.627$

$J = At^2/3 = 1.94(.25)^2/3 = 0.04042\ in.^4$

$I_w = 2(3.04) - 1.226 = 4.854;$ $\quad r_w = \sqrt{\frac{4.854}{1.94}} = 1.58"$

Consider $K_w = 0.8$

$F_{ew} = \pi^2(29000)/(.8(64)/1.58)^2 = 273$ ksi

$F_{ej} = (11{,}200)(.04042)/(1.94(4.996)) = 46.7$ ksi

$$F_e = \frac{273 + 46.7}{2(.627)}\left[1 - \sqrt{1 - \frac{4(273)(46.7)(.627)}{(273 + 46.7)^2}}\right] = 43.6 \text{ ksi}$$

$\frac{29{,}000}{43.6} = 81 > 80.5$

$\frac{81}{132.1} = .613,\ C_a = .436$

$F_a = .436\ (.911)(36) = 14.3$ ksi

$f_a = 7/1.94 = 3.61$ ksi

$M = P\ (1.45) = 7(1.45) = 10.15^{"k}$

$f_{bz} = 10.15(1.375)/1.226 = 11.38$ ksi

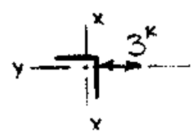
$F_{bz} = .911\ (.60)(36) = 19.68$ ksi

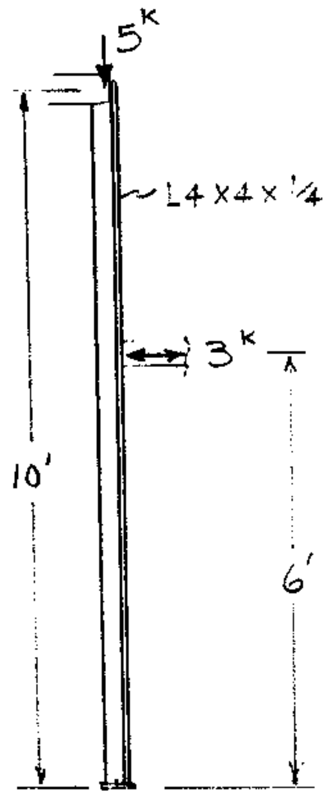
$F_e' = 22.76$ $\quad C_m = 1$

$$\frac{3.61}{14.3} + \frac{(1)11.38}{19.68(1-3.61/22.67)} = .252 + .688 = .940 < 1 \quad \text{OK}$$

Example Problem 2

Consider a L4x4x1/4 used as one of the support legs of a machinery platform. This leg, which is part of a braced framework supporting the platform, can be considered as an axially loaded strut with a 10' unbraced length. At 6' along this length another smaller platform uses this support leg as a lateral support in the angle's y-direction. Under dead plus seismic load the angle is loaded as shown at right. First check the adequacy of this angle unbraced over the 10' length.





$$f_a = 5^k/1.94 = 2.58 \text{ ksi}$$

$$KL/r = (1)(120)/.795 = 151$$

$$F_a = 6.55 \text{ ksi}$$

$$M = 3(4)(6)/10 = 7.2^{"k}$$

Since the strut is laterally unbraced, from Eq. 14

$$f_b = 1.25(7.2)/1.05 = 8.57 \text{ ksi}$$

$$C_b = 1; \; Lt/b^2 = 120(.25)/(4)^2 = 1.875 < 2.43$$

F_{obx} need not be evaluated; however, evaluate anyway.

$$F_{obx} = \frac{85900}{(120/4)^2} (1) \left[\sqrt{1 + .78(1.875)^2} - 1 \right] = 89.19$$

$$\sqrt{F_y/F_{ob}} = \sqrt{36/89.19} = \sqrt{.4036} = .635$$

$$F_b = 22.76 \text{ ksi from Eq. 11b}$$

From local buckling $F_b = .60QF_y = .6(.911)(36) = 19.68$ ksi

$$r_b = r_x/1.35 = 1.25/1.35 = .926$$

$$KL/r_b = (1)120/.926 = 129.6; \; F_e' = 8.89 \text{ ksi}$$

$$\frac{2.58}{4/3(6.55)} + \frac{8.57}{19.68(4/3)(1 - 2.58/(4/3(8.89)))} = .295 + .418 = 0.713 < 1 \text{ OK}$$

If the strut is laterally braced in the x-direction at the point at which the 3 kip load is applied, the angle is able to bend and buckle in the y-direction (about the x-axis) only. The brace used should be relatively stiff and able to take $.6(3) = 1.8^k$ of lateral for the following calculation to be valid.

For 10' length, $KL/r = (1)(120)/1.25 = 96$ using r_x

For 6' length using $K \leq 1$, $r = r_z$; $KL/r = (1)(72)/.795 = 90.6$

$96/132.1 = .727$, $F_a = .389(.911)(36) = 12.76$ ksi

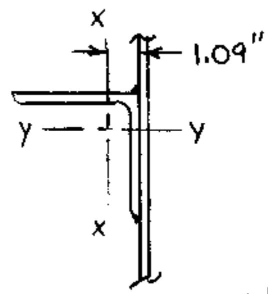
$f_b = M/S_x = 7.2/1.05 = 6.86$ ksi

From previous evaluation of F_{obx} it is obvious that local buckling will control the determination of F_b = 19.68 ksi. However, if F_{obx} were evaluated, L would be 72" and C_b would be 1.5 with M_1/M_2 = 0.

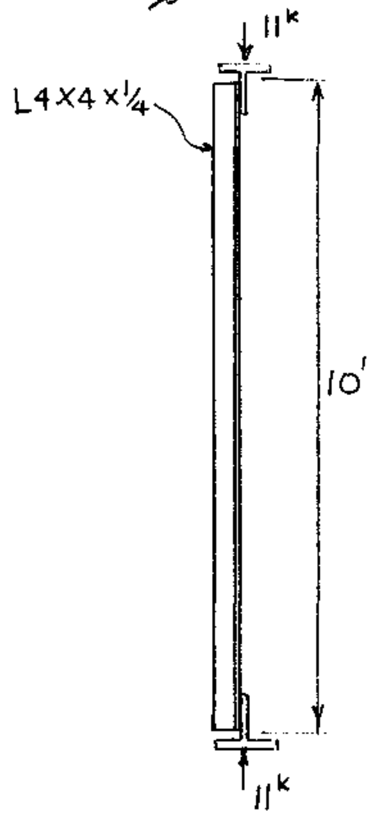
$KL/r_b = KL/r_x = 96$, $F'_e = 16.2$ ksi

$$\frac{2.58}{4/3(12.76)} + \frac{6.86}{4/3(19.68)(1-2.58/(4/3 \times 16.2))} = .152 + .297 = 0.449 \quad \text{OK}$$

<u>Example Problem 3</u>



Check a L4x4x1/4 diagonal strut with an unbraced length of 10' for a compressive load of 11^k. Consider one leg welded to the stem of a WT chord member at each end such that K_y can be considered as 0.65 and the chords laterally braced such that K_x can be considered as 1.0
F_y = 36 ksi.



Use $r_{eq} = .88 r_x$ from Fig. 3 which is more conservative than use of $r_{eq} = r_x$ (7,8)

$$\frac{KL}{r} = \frac{10(12)}{.88(1.25)} = \frac{120}{1.10} = 109$$

Since Q = .911 with b/t = 16, $C_c = 132.1$

$109/132.1 = .825$; $F_a = .346(.911)(36) = 11.35$ ksi

$f_a = 11/1.94 = 5.67$ ksi

Research (7,8) has indicated that bending occurs primarily about the geometric x-axis for this situation such that design for flexure can be based on geometric axis bending of the angle. Consider the load applied at the weld, so that the eccentricity of load equals $\bar{y}$ = 1.09".

$$f_b = \frac{(P\bar{y})\bar{y}}{I_x} = \frac{11(1.09)^2}{3.04} = 4.30 \text{ ksi}$$

Since $b/t = 16 > 76/\sqrt{F_y}$, $F_b = .60QF_y = 19.68$ ksi with $Q = .911$ from Eq. 3b.

$r_b = r_x = 1.25"$

$$\text{so } F'_e = \frac{12\,\pi^2 E}{23(120/1.25)^2} = 16.2 \text{ ksi}; \quad C_m = 1$$

$$\frac{5.67}{11.35} + \frac{4.30}{19.68(1-5.67/16.2)} = .500 + .336 = 0.836 < 1$$

Comparing these results with those of Ex. Prob. 1, one can see that significantly more compression load can be taken on a longer unbraced length with an angle strut with one leg attached to the chord stem.

If in tension, this strut would be capable of taking the smaller of $A_e F_b$ = .85(1.94)(.5x58) = 47.8 kips if ends welded with a combination of longitudinal and transverse welds or 1.94(.6)(36) = 41.9 kips.

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