Ordinary meeting

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Geotechnical design of retaining walls

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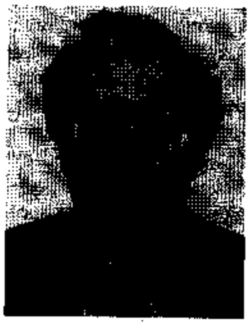

Malcolm Bolton graduated from Cambridge in 1967, and took a MSc researching the 3-D behaviour of structures at Manchester University. In 1969 he went to UMIST to help set up the UK's first geotechnic modelling centrifuge. He remained at UMIST as a lecturer until 1979, writing his textbook A guide to soil mechanics. *He returned to Cambridge University in 1980 where he is now Reader in Geotechnical Engineering and Director of the Geotechnical Centrifuge Centre.*

Synopsis

The safe and economic design of a retaining wall depends on the appropriate mobilisation of strength in the adjacent soil. Dense soil tends to be brittle, so that it loses its strength even under strains compatible with the expected displacement of walls. Loose soil tends to be so compliant that it fails to fully develop its available strength. These two concerns are central to the new design methodology adopted by BS8002 Code of practice for earth-retaining structures.

Examples are given of the selection of design values for the strength of a variety of soils. The reasons for suspicion regarding the cohesion of clays are expounded. An extended example is then provided of the design of a simple cantilever retaining wall, following BS8002. It is shown how to construct a free-body diagram in equilibrium with design earth pressures. This is then used to derive a first estimate of bending moments in the wall. Some circumstances are then discussed in which the moment of resistance should be increased. However, it is demonstrated that the equilibrium condition places a strict upper bound on any such increase.

The concept of mobilisable soil strength offers a logical and scientific basis for the design of all geotechnical structures. It can be seen as satisfying all the objective requirements of the limit state method with none of the attendant difficulties of the partial factor format.

Introduction

The new Code for earth-retaining structures[1], BS8002, is quite revolutionary but at the same time it is quite elementary to apply. The radical step has been to eliminate the various safety factors against sliding, overturning, etc., which were used in CP2 (1951)[2], together with the safety factor of 2 on passive pressure. Instead, there is simply advice on the derivation of a design soil strength which satisfies both safety and serviceability and on the creation of a design scenario which features worst-credible loads and the most unfavourable environment that could reasonably be assumed for the proposed structure. All the designer then has to do is to satisfy global and then local equilibrium of the structure under the action of earth pressures which correspond to the design soil strengths.

This approach is set out as a limit state design method, but it can also be seen as a permissible stress method. Since the design strength will never exceed the peak soil strength reduced by a factor M, adherents of partial factors can imagine that M is a partial factor. Those who insist on using the term 'safety factor' can see M as a safety factor on soil strength. In fact M is a mobilisation factor which has been derived from the need to control displacements[3]. Those who make non-linear finite element analyses of retaining walls may like to check that a wall designed using the advisory value of M does not displace by more than 0.5% of its height in moderate to good soils.

The explicit treatment of equilibrium and deformation are the keystones of the new design method. The approach can be recommended for the design of all classes of geotechnical structure or foundation, if the magnitude of permissible displacement is carefully reflected in the mobilisation factor. The aim of this paper is to set out these new-style calculations so as to demonstrate their ease of use. Designers should note in particular that the new approach leaves them free to apply whatever earth pressure coefficients, mechanisms, or methods of plastic analysis they may find preferable in any particular application. However, those wishing to update from CP2 (1951) to BS8002 (1994) must equally update their basic soil mechanics; this is treated first.

Density of soil

The density of soil ρ depends on its voids ratio e, the degree of saturation of its voids S_r from 0 (dry) to 1 (zero air voids), and the specific gravity G_s of its grains. The bulk density can then be expressed in relation to the density of water $\rho_w = 1000$ kg/m^3.

$$\rho = \rho_w \cdot (G_s + e\, S_r)/(1 + e) \qquad \text{....(1)}$$

For free-draining soils, the term relative density I_D is also used to indicate the position of the current voids ratio on a linear scale from the densest achievable after vibration (e_{min}, $I_D = 1$) to the loosest achievable by quick slumping (e_{max}, $I_D = 0$). For fine-grained soils, the water content remains constant during sampling and it can be related to the voids ratio by the expression $w = e\, S_r/G_s$.

Strength parameters for soils

Soil comprises grains and voids. It may be regarded either as a single-phase material or as a composite material for which the grains and voids are dealt with separately.

The shear strength c of a mixed material like concrete or epoxy cement is usually quoted as a given 'cohesive' strength which is the maximum possible shear stress which can be induced:

$$c = \tau_{max} \qquad \text{....(2)}$$

It is recognised that c will depend on the particular mixture, and in the case of soil it is found that the voids ratio is of first importance:

$$c = f(e) \qquad \text{....(3)}$$

though more careful inspection reveals that preconsolidation pressure and rate of shearing also have some influence. Unlike concrete, the voids in soil are not fixed in place; water can move and voids can collapse or expand. This is such a strong feature that engineers have become uncomfortable using the cohesive material model for soil except with fully saturated, uniformly fine-grained clay in the short term, when its relative impermeability keeps its voids ratio constant[4].

Modern composite materials, such as fibre-reinforced plastic, are often treated as dual-phase with their components first considered separately and then superimposed. This echoes Terzaghi's earlier treatment of soil as a two-phase material with separable 'effective stresses' (σ', τ') and 'pore pressures' u, carried by the aggregate skeleton and its voids. In this view, 'total stresses' (σ, τ) due to gravity can decomposed thus:

$$\begin{array}{ll} \text{normal stress} & \sigma = \sigma' + u \\ \text{shear stress} & \tau = \tau' \end{array} \qquad \text{....(4)}$$

The simplest idealisation of the shear strength of a granular aggregate is in terms of the angle of internal friction ϕ_{max}, where

$$\tan \phi_{max} = (\tau / \sigma')_{max} \qquad \text{....(5)}$$

but it has to be recognised that internal friction depends not only on interparticle friction but also on the degree of particle interlocking. Interlocking leads both to dilatancy and to an extra component of internal angle of friction $\Delta\phi$. Soil shears at constant volume under particular conditions of high stress or low density known as 'critical states' for which the angle of friction can be regarded as a constant, ϕ_{crit}. In general conditions of shearing with dilatancy, the two components of internal friction both contribute:

$$\phi_{max} = \phi_{crit} + \Delta\phi \quad ....(6)$$

The dilatant component of friction carries almost all the potential uncertainty for the designer since it can be as high as 20° for the densest packing of rigid angular grains, but can reduce to zero if there is a reduction either in the degree of compaction or in the strength of the grains relative to the imposed stresses. In other words,

$$\Delta\phi = f(e, \sigma') \quad ....(7)$$

as expressed in Bolton[5]. Since dilatancy is irreversible, like a ratchet and pawl, it also follows that $\Delta\phi \Rightarrow 0$ at large strains. For example, it may take a granular material of the order of 2% strain to reach peak strength with ϕ_{max}, but it will then take a shear displacement of only about 5 particle diameters on a slip plane for the friction to drop to ϕ_{crit}.

Certain presumed values of internal friction are given in BS8002, section 2.2, Tables 2, 3, and 4. They should be replaced by measurements wherever this is practical. For granular materials, which may be difficult to sample, Table 3 breaks down the components of friction angle thus:

$$\phi_{crit} = 30 + A + B \quad ....(8)$$

$$\phi_{max} = \phi_{crit} + C \text{ (i.e. } C \equiv \Delta\phi) \quad ....(9)$$

where A and B are components of the basic critical state friction due, respectively, to angularity and grading and C is the maximum possible dilatant component based on a SPT blow-count corrected for stress level.

It is important to realise that the shear strength of soil (any soil, at any time) can be estimated either following (2) while allowing for (3), or following (4, 5, 6) while allowing for (7, 8, 9). It is the relative ease of making the appropriate allowances in different circumstances which has led to the short cut of reserving the word 'undrained' for the short-term cohesive strength c_u of clay at constant voids ratio and the word 'drained' for the long-term frictional strength of any soil whose pore pressures are no longer a function of the loading (so that they can be estimated easily from a hydraulic analysis).

The angle of internal friction must then be recognised to be a variable. BS8002 is the first UK Code of Practice to specify that designers must not depend on dilatancy in soils. For safety:

$$_{design}\phi \leq \phi_{crit} \quad ....(10)$$

This was judged proper since:

- walls often support or rely on natural soils whose density is uncertain
- fully softened slip surfaces may develop in the soil before the wall is completed
- accidental movements may occur because of flooding, loading, or excavation at the toe

A retaining wall properly designed to the new Code would behave as a plastic, ductile structure even if it were subjected to excessive accidental loading. This harmonises with the structural engineer's approach, following Ronan Point, to the design of walls in buildings which may be asked to survive the lateral pressure of explosions. Reliance on brittle, discontinuous behaviour is widely accepted as bad structural practice.

Total stress analysis v. effective stress analysis

Following the coining of the term 'effective stress analysis' to describe the treatment of soil as a two-phase material, the term 'total stress analysis' came to be used for the single-phase treatment in which pore water pressures simply do not appear. It would have been much better to refer to 'cohesion' and 'friction' models of behaviour, as indicated in eqns (2) and (5), respectively. Engineers have become confused about the fact that both models can be applied simultaneously to all soils. They let this confusion cloud their judgment regarding which model to chose in any particular set of circumstances. The art is simply to select a material idealisation whose parameter can be estimated with least uncertainty.

Any tendency for the pore water to drain externally will lead to changes of voids ratio and therefore to changes of 'cohesion' which are difficult to predict quantitatively. For this reason the drained strength of soil is almost invariably assessed using effective stresses and friction. Any tendency to drain internally should lead to the same conclusion, but with greater danger since the time for transient flow (swelling, in this case) is much reduced since the drainage paths are much shorter. This happens with stiff, over-consolidated soils such as London clay which indulge in brittle shear rupture forming wettened or softened zones of sliding within the mass, long before Terzaghi's consolidation theory would have predicted. Whenever the undrained strength of a soil exceeds its drained strength it is vulnerable to drainage, whether through preexisting sand layers or fissures, load-induced shear ruptures or tensile cracks. Understood in terms of pore pressures, all such cases can be seen as due to the quicker-than-anticipated relaxation of temporarily reduced pore pressures.

It is wrong to risk human life on the hope that suction can be maintained in fine-grained soils. What is satisfactory for the tax-disc on the windscreen, or the rubber cup which pegs a towel to the kitchen wall, is unsatisfactory for a retaining wall – mainly because the rubber cup or plastic disc is manufactured specifically to maintain suction while the ground is full of undiscoverable flaws (and leaky sewers, water mains, etc.). For this reason, the undrained strength of clay in a 'total stress' analysis should be allowed to rule in design only when it is inferior to the drained friction analysis which must always accompany it. This occurs with lightly over-consolidated clay (i.e. mud). Otherwise, clays which are firm or stiff will appear stronger in undrained strength calculations: they must be set aside, and replaced by drained calculations based on internal friction and conservative water pressures[6]. BS8002 expounds this philosophy at length, and in various sections, including 2.2.3, 3.3.3 and 3.3.5.

Mobilisation of strength

Good granular soil is less stiff than concrete by a factor of between 10^3 and 10^4. Nevertheless, structural engineers have been slow to recognise that the compliance of soil, expressed as the strain required to mobilise its strength, is actually the controlling factor in the design of earth-retaining structures.

It is easily demonstrated[7] that a wall which rotates by 1/200 mobilises average compressive and tensile strains of $\pm 0.5 \times 10^{-2}$ in adjacent earth, corresponding to an average shear strain of 1%. BS8002 took the view that larger wall rotations would not generally be acceptable, and therefore set out to limit the design strength of soils to that which could be mobilised at 1% shear strain. This is a similar approach to that often adopted for ductile alloys, where a 0.2% proof stress (for example) means a design stress which mobilises 0.2% strain in a standard test. This does not preclude more careful analyses of deformation; nor does it imply that larger strains are excluded at every point in the material simply by introducing a blanket restriction on design stress. What it does achieve is the setting of a standard for the maximum mobilisation of strength which is consistent with the maximum permissible deformation of the structure as a whole.

In the future, engineers must be encouraged to specify and use stress-strain tests to determine the permissible mobilisation of strength. For the present, BS8002 simply specifies a peak strength reduction factor M, called the 'mobilisation factor', which aims to satisfy the 1% shear strain criterion. Following some inspection of data, it was decided that soils which were at least 'medium dense' or 'firm' should be designed to permissible stresses using a value $M = 1.5$ against eqn (3), and a value $M = 1.2$ against eqn (5). Softer soil is said to require a greater mobilisation factor; this would have to be selected from a stress-strain test mobilising 1% shear strain (or some other magnitude corresponding to the desired control of wall displacements). For serviceability:

$$_{design}\tau \leq \tau_{max}/M \quad ....(11)$$

Engineers who insist on having a 'factor of safety', but who fail to take soil strains into account, must simply realise that the one compensates for the lack of the other. This has been explained previously by the author[8]. BS8002 makes this explicit, reserving the word 'safety' to problems of collapse which involves ϕ_{crit} and using the concept of 'serviceability' for the control of deformations by means of a mobilisation factor on $\tan \phi_{max}$ (taking the effective stress analysis of sands as a typical example).

This is a design decision, not a deformation analysis. If it were desired to predict wall displacements very accurately, it would be necessary to perform a numerical analysis with some non-linear soil-structure interaction package and some carefully selected stress-strain curves. The initial earth pressures, the method of wall installation, and the construction sequence, would all be highly significant determinants of the final wall displacements. These issues are mentioned, but are not detailed, in BS8002. Walls designed to BS8002 should nevertheless displace by no more than about 0.5% of their height; this is a general feature of the plastic mobilisation concept[9].

Establishing strength parameters for various soil types

It will helpful, in order to demonstrate the new approach, to take some examples.

Soil type A: well-graded granular fill, compacted in layers by a method which guarantees at least 92% of Proctor optimum density. For a typical specific gravity $G_s \approx 2.66$, and maximum and minimum voids ratios $e_{max} \approx 0.8$ and $e_{min} \approx 0.5$ which are typical for such fill, the corresponding densities from (1) would be:

TABLE 1 – Typical densities of granular fill

I_D = ρkg/m³	0.000 loosest	0.33 loose-medium	0.66 medium-dense	1.00 densest
Dry	1478	1565	1663	1773
Saturated	1922	1976	2038	2100

If the Proctor density test were to achieve the maximum relative density, the field-compacted dry density would be 92% of 1773kg/m³, i.e. 1631kg/m³. However, experience proves that the standard laboratory compaction test does not quite achieve this. A designer might instead rely on achieving only a relative density $I_D = 0.66$ in the field, with a dry density of 1565kg/m³ and a saturated density of 2038kg/m³. A well-graded fill is likely to contain enough fines to retain a high capillary water content even when drained. The design value of unit weight would then be $\gamma = 2.038 \times 9.81 = 20.0$kN/m³. Although there will always be some uncertainty in the unit weight of fill as placed, it will generally be possible to select a reasonable and conservative value.

BS8002, Table 3, gives guidance on the three contributions to the expected angle of internal friction. We may assume the fill is subangular and take $A = 2°$, and we are also given $B = 4°$ for a well-graded aggregate. Part C lists four progressively increasing strength contributions, attributable to the dilatant interlocking of dense grains. The medium-dense compacted fill corresponds to the third case, equivalent to a corrected SPT of 40, and gives $C = 6°$. Eqns (8) and (9) therefore lead us to presume that:

$$\phi_{crit} = 30 + A + B = 36°, \text{ and } \phi_{max} = \phi_{crit} + C = 42°.$$

A more rigorous search through the available database (e.g. Bolton[5]) would show that these values are likely to underestimate the angle of internal friction of moderately compacted granular materials at the low stress levels usually found in earth retention.

For design, safety then dictates that $_{design}\phi \leq \phi_{crit}$ which is 36°, and serviceability dictates that $_{design}\phi \leq \tan^{-1}\{(\tan \phi_{max})/1.2\}$ which is 36.9°. This well-compacted fill is safety-limited and must be designed to mobilise $\phi = 36°$.

Soil type B: a natural fine, dune sand, with uniform rounded grains, found from SPTs to be generally of medium density, but with loose-medium pockets (corrected blow-count $N' \approx 20$). Maximum and minimum voids ratios of a uniform sand would be higher than for a well-graded fill, but rounded sands also trap less voids than angular sands. The engineer would allow for less water retention above the water table, and for slightly reduced density below the water table, compared with soil type A in Table 1. The design value of the saturated unit weight might be taken to be $\gamma = 19.5$kN/m³.

Using $A = 0$ and $B = 0$ in BS8002, Table 3, gives $\phi_{crit} = 30°$. Using $C = 2$ gives $\phi_{max} = 32°$. We then establish that $_{design}\phi \leq \tan^{-1}\{(\tan 32°)/1.2\}$ which is 27.5°. This sand is deformation-limiting, with $_{design}\phi = 27.5°$. The mobilisation factor of 1.2 is good only for sands of medium density and cannot be guaranteed to control deformations in loose sands. It has to be recognised, for example, that, if there were strong ground vibrations in service, they may lead to compaction and subsidence. If there were occasional loose layers, rather than occasional pockets, this would cause even more anxiety. Vibrocompaction of the sand prior to construction would be a good alternative option for sensitive structures such as bridge foundations, for example. If medium to good density could then be guaranteed, $C \Rightarrow 6°$, so $\phi_{max} \Rightarrow 36°$, which converts the situation to a strength-limiting design with $_{design}\phi = \phi_{crit} = 30°$.

Soil type C: a natural firm to stiff, glacial clay with frequent silty and sandy laminations, a natural water content close to the plastic limit of 15%, and a liquid limit of 30%, these Atterberg limits being determined for the most clayey material. Since the plasticity index is 30% – 15% = 15%, BS8002, Table 2, suggests a presumed value for $\phi_{crit} = 30°$. The ultimate friction angle recorded after large displacements in a direct shear test on a submerged clayey sample, sheared very slowly so that it is freely drained, is one practical method of obtaining ϕ_{crit} in the laboratory. This critical state angle of friction sets an upper bound to the design angle of friction to be used in a drained stability analysis.

It should be much more common for engineers to demand undrained triaxial tests with pore-pressure measurement for clay samples and to ask for the measurement of axial strain ε over an internal gauge length. It would then be possible to confirm that $\phi_{\varepsilon = 0.5\%} \geq 30°$ by drawing a Mohr circle of effective stress at the serviceability limiting strain. An experienced geotechnical engineer who was also a risk-taker might accept, without checking, that this would be the case for an over-consolidated low-plasticity clay at the plastic limit, and might therefore feel free to use 30° in design.

A generalist who was not prepared to get samples tested would have to use the presumed value $\phi_{crit} = 30°$, and would have to neglect the possible dilation at peak strength so that he or she would assume that $\Delta\phi = 0$, and therefore that $\phi_{max} = 30°$. This is equivalent to the $c' = 0$ assumption when using strength envelopes; any strength above a lower-bound ϕ_{crit}-line is ignored. If the peak strength were only 30°, a smaller value must obviously be used in design. The generalist would apply the Code's mobilisation factor to obtain $_{design}\phi = \tan^{-1}\{(\tan 30°)/1.2\} = 25.7°$. Of course, the assumption that a M value of 1.2 protects sufficiently against soil mobilisation would have been unjustified if the clay had been described as soft.

Undrained strength calculations are generally unsafe for retaining walls since drainage can occur. Suppose that the SPT blow-count for the clay was 15 to 30. Empirical correlations then suggest that the undrained shear strength may be 75 to 150kPa. However, the laminated soil fabric reported earlier would strongly indicate that no such interpretation be used, and certainly not where it indicated a smaller active pressure, or a larger passive pressure, than that which could be derived on the basis of completely drained behaviour.

Design earth pressures: effective stress analysis

Earth pressures acting on a wall due to surcharge q and unit weight γ in the neighbouring soil are calculated using effective stress analysis as follows:

- find the nominal vertical stress at a point, $\sigma_v = q + \gamma z$, so that $\sigma_v' = \sigma_v - u$
- declare whether the ground at that point is tending to subside and spread laterally (active mobilisation) or to heave following lateral constriction (passive mobilisation)
- select an earth pressure coefficient K (K_a for active mobilisation or K_p for passive mobilisation), which is a function of the selected internal angle of friction of the soil $_{design}\phi$ and a selected value of the angle of friction which can be mobilised against the wall $_{design}\delta$
- calculate the stress on the wall, $\sigma = u + K(\sigma_v - u)$

Usually, walls will be rough, in the sense that their surface texture exceeds the mean particle size. In those circumstances BS8002 instructs the designer to assume that $_{design}(\tan\delta/\tan\phi) \leq 0.75$. For a typical granular fill (soil type A above) with $_{design}\phi = 36°$, this gives $_{design}\delta \leq 28.6°$. Formulae, tables or charts of the designer's choice can then be entered to interpolate for K_a or K_p at $\delta/\phi \leq 0.8$. Conservative values are found by using Rankine's coefficients based on zero wall friction. Unconservative values are provided by Coulomb's wedge mechanisms. The most reliable values are found from applications of the method of characteristics; tabulations on a similar basis are found in Kerisel & Absi[16]. Limiting values of the order of 0.21 and 8.7, respectively, will be obtained for a vertical wall retaining horizontal fill in this case, so that $_{design}\sigma_a' \geq 0.21\,\sigma_v'$, and $_{design}\sigma_p' \leq 8.7\,\sigma_v'$.

Design earth pressures: total stress analysis

Earth pressures acting on a wall due to surcharge q and unit weight γ in the neighbouring soil are calculated using total stress analysis as follows:

- find the nominal vertical stress at a point, $\sigma_v = q + \gamma z$
- declare whether the ground at that point is tending to subside and spread laterally (active mobilisation) or to heave following lateral constriction (passive mobilisation)
- select an equilibrium factor N which is a function of the proportion α of the cohesive strength c_u of the soil which can be mobilised on the face of the wall, where for a simple vertical wall against a level stratum of clay N takes values from 2.00 to 2.57 as α increases from 0 to 1
- calculate the stress on the wall, $\sigma = \sigma_v \pm Nc_u$; + for passive, – for active

The approach given in BS8002 amounts to the same procedure.

Design

The only requirement of BS8002 is that walls should be shown to be in equilibrium under the action of permissible earth pressures no more extreme than those calculated as design earth pressures. Safety and serviceability is delivered by the selection of the value of design soil strength, from which earth pressures have been deduced. At the same time, water tables are to be set as high as would be reasonable, a surcharge of at least 10kPa is generally

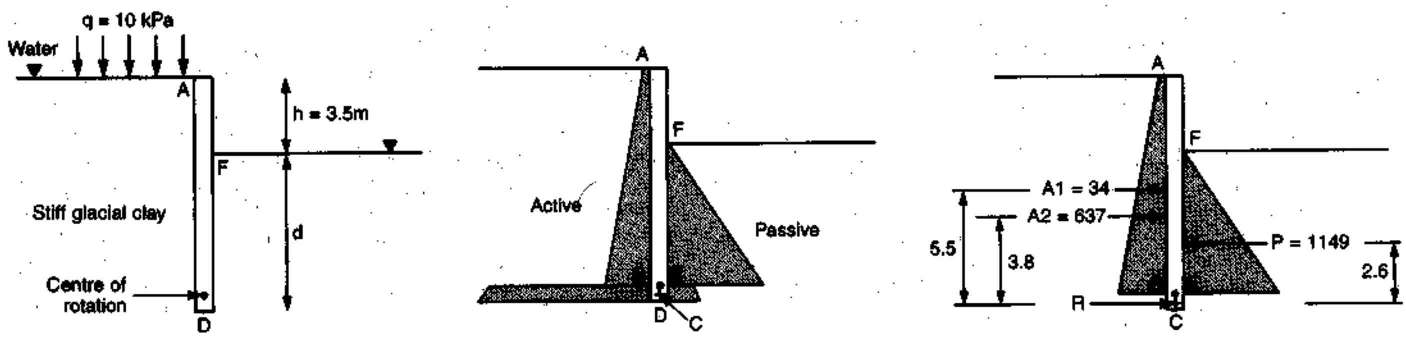

Fig 1 (a). Design cross-section of cantilever *Fig 1 (b). Active and passive pressure* *Fig 1 (c). Approximate equilibrium free- body*

to be placed on the retained fill to simulate traffic, and an over-dig of at least 0.5m is to be assumed in excavations. No further safety factors are then appropriate. The wall must simply be shown to have an equilibrium free-body diagram.

Example: cantilever wall in stiff glacial clay

Design situation

Consider a simple cantilever wall to retain a 3m cutting in glacial clay of soil type C, described above. The cutting might be for a road, and the wall might be of steel sheet-piles or bored cast-in-place concrete, for example. To conform to the design situation imposed in BS8002, section 3.2.2.2, the retained height will be increased to 3.5m and the retained surcharge will be 10kPa. It will be assumed that the preexisting water table was high, so the phreatic surfaces in the long term will be taken at the levels of the ground surface each side of the wall, conforming to 3.2.2.3. There would be a distinct advantage in permitting drainage through the wall, but this might be regarded as requiring specialist geotechnical attention, and the objective here is to set out some simple calculations for the non-specialist.

Fig 1(a) shows the design cross-section. Information given above for soil type C can be used to establish the following values: ${}_{\text{design}}\gamma = 21\text{kN/m}^3$, ${}_{\text{design}}\phi = 25.7°$, and so ${}_{\text{design}}\delta = 19.8°$. Earth pressure coefficients $K_a = 0.33$ and $K_p = 4.2$ can then be found from the tables of Kerisel & Absi. It remains to guess the wall depth necessary to support a 3.5m face under these circumstances, so that this may be iterated until equilibrium is achieved.

Global equilibrium: initial trial

The approach of Bolton, Powrie & Symons[11] offers a starting point. Their Fig 2 shows that a cantilever wall in soil with $\gamma/\gamma_w = 2$, with full height phreatic surfaces, and mobilising the required $\phi = 25.7°$, $\delta/\phi = 0.77$, needs an embedment ratio $d/h = 2.2$. For $h = 3.5$m, we get $d = 7.7$m, so a total wall height of 11.2m is indicated. Our soil is heavier, which adds stability, but it is also surcharged on the active side, which reduces stability; a wall height of 11.0m will therefore be our starting point. Their figure also shows that such a wall would pivot about a point 7% up from the toe of the wall, which is about 0.8 m. Earth pressures flip over at the pivot from active to passive: see Fig 1(b) where the pivot is represented by point B on the retained side and E on the excavated side. Equilibrium will be considered to have been sufficiently well proven if all the earth pressures above BE are shown to be in moment equilibrium about point C, which is at the midpoint of the 'fixed–earth' zone BD between the pivot and the toe.

It is now necessary to calculate earth pressures at A, B, E and F. The first step is to find the pore pressure at D assuming a linear rate of piezometric pressure reduction around the wall (BS8002, section 3.3.5.2). In this case,

$$u_D = 9.8 \times 11\,[1 - 3.5/18.5] = 87\text{kPa}$$

Since it is assumed that pore pressures change linearly we can deduce

$$u_B = 87 \times 10.2 / 11 = 81\text{kPa}$$
$$u_E = 87 \times 6.7 / 7.5 = 78\text{kPa}$$

The earth pressures calculated in Table 2 lead to the pressure diagram in Fig 1(b). This leads to the calculation of active forces $A1 = 34$kN/m and $A2 = 637$kN/m and passive force $P = 1149$kN/m, with magnitudes derived from areas on the pressure diagram, and lines of action passing through the centroids of those areas, as shown in Fig 1(c). They are to be considered to be held in equilibrium by a fixing force R acting at C. Fig 1(c) is a trial equilibrium free–body diagram constructed on the assumption that the wall should be 11m deep to resist design earth pressures. The first step in checking global equilibrium is to take moments for all forces about C.

TABLE 2 – Earth pressures calculated at salient points in Fig 1

Stresses kPa	u	$\sigma_v = q + \gamma z$	$\sigma_v' = \sigma_v - u$	$\sigma_h' = K\sigma_v'$	$\sigma_h = \sigma_h' + u$
A	0	10	10	3	3
B	81	224	143	47	128
F	0	0	0	0	0
E	78	141	63	265	343

Overturning moments are: $34 \times 5.5 + 637 \times 3.8 = 2608$kNm/m
Restoring moments are: $1149 \times 2.6 = 2987$kNm/m

Satisfying global equilibrium

Whereas in previous styles of safety audit the restoring moments should have exceeded the overturning moments by some safety factor, BS8002 has already dealt with safety by this juncture. We are now in the middle of a routine piece of structural analysis for a wall. It should be in equilibrium, so the overturning moments should exactly balance the restoring moments.

It is evident that, if 379kNm/m of extra overturning moment were applied, the 11m wall would be in equilibrium. It is marginally too deep for its current requirements, if the full design soil strengths are to be mobilised. One way of controlling the subsequent iteration which the designer may wish to pursue is to calculate the actual surcharge which would bring the wall into equilibrium at its current depth. This is easily computed; the force $A1$ should be increased by 379/5.5 = 69kN/m, so the active pressure should be increased by 69/10.2 = 6.8kPa, so an additional surcharge could be carried equal to 6.8/0.33 = 21kPa. The 11m wall can carry a surcharge on the retained soil of 31kPa. If desired, the depth could speculatively be reduced by 1m and the calculation repeated. Interpolation between equilibrium values of applied surcharge would then indicate a wall depth of 10.5m which almost exactly mobilises the permissible soil strength under an active surcharge of 10kPa.

Once the wall is in moment equilibrium, other forces and stress–resultants can be calculated. Suppose that we accept the current situation of the 11m wall. Above BE, the new active forces are 740kN/m, the passive forces are 1149kN/m, so the toe resistance $R = 1149 - 740 = 409$kN/m. This is supposed to be developed over the bottom 0.8m, if the original assumption about the pivot point is valid. The net resisting pressure below the pivot BE is therefore required to be 409/0.8 = 511kPa. To check whether this is mobilisable, a simplified approach can be used for this small section near the toe. The vertical effective stress at B and at E can be used to find conservative estimates of the mobilisable passive pressure on BD and active pressure on ED, as shown in Table 3.

The net resisting pressure is therefore at least 770 – 99 = 671kPa, which exceeds the requirement of 511kPa. The 'fixed-earth' region is well fixed. As with the over-provision of restoring moment, it is not essential to mobilise all the available soil strength. In this case, the designer should probably be satisfied to have a design for a wall in equilibrium in which every soil zone is approaching its permitted degree of mobilisation under a slightly more onerous design situation than that set by the Code.

Calculating bending moments

Once a free-body diagram has been produced, bending moments can be calculated. The 11m-deep wall, carrying 31kPa of surcharge on its retained soil, has a mobilised total active pressure of 10kPa at the retained surface, rising at 12.25kPa/m, and a mobilised total passive pressure rising at 51.2kPa/m below the excavation. Net pressure, shear force and bending moment diagrams are easily drawn. Shear force is found to pass through

TABLE 3 – Earth pressures calculated at salient points in Fig 1

Stresses kPa	u	$\sigma_v = q + \gamma z$	$\sigma_v' = \sigma_v - u$	$\sigma_h' = K\sigma_v'$	$\sigma_h = \sigma_h' + u$
B to D	81	245	164	689	770
E to D	78	141	63	21	99

zero at a depth of 4.1m below the design excavation; this offers a maximum bending moment of 597kNm/m at that elevation. This is the bending moment which would occur if the earth pressures were exactly what had been calculated. This, of course, will not actually be the case. So, should this design bending moment be factored-up in some way to derive the design bending resistance for the cross-section of the wall?

Should the design bending resistance be equal to the design bending moment?

First, consider the degree of safety already built in to the calculations.

(1) The stiff clay has been allowed to become fully softened so that it can be treated as a frictional material. This tendency, referred to in vague terms in CP2, has been calculated above with some care and consistency. The degree of mobilisation of frictional strength was, however, the arbitrary value recommended in the new Code.

(2) The water table has been allowed to remain at the retained soil surface, instead of being drawn down by the excavation. This assumes steady flow; on the other hand, the clay has layers of sand which can conduct water towards the wall from the retained land, so this is not implausible.

(3) The excavation has been over-dug by 0.5m, and a surcharge in excess of 10kPa has been applied on the retained side, and these have been assumed to occur for long enough to be reflected in the fully-softened frictional soil strength.

It seems that the only possible justification for increasing the design bending resistance would be where the designer conscientiously anticipated that the degree of strength mobilisation might differ from that assumed earlier. However, there is an extremely powerful constraint on the degree to which this could lead to changes in bending moment. The wall must remain in equilibrium, and the general form of the earth pressure diagram must remain similar. In particular, the wall must still be in moment equilibrium about some point C close to its toe, at which the 'fixed-earth' resistance can be considered to be concentrated. So if the 'active' pressures increase by 10%, the 'passive' pressures must also increase by 10% to achieve moment equilibrium; only then can the bending moments increase by 10%. It is this constraint which other Codes have failed to apply. Equilibrium is a stern task-master, but a powerful ally against uncertainty.

There are logical limits to the mobilisation of larger earth pressures. On the retained side, the soil would eventually approach an earth pressure coefficient of unity as its mobilised angle of friction approached zero. On the excavated side, the soil would quickly attain its full passive pressure. Consider the use of ${}_{design}\phi = 30°$ on the passive side of the wall, which gives a fully mobilised passive earth pressure coefficient of about 6.0. Using this in Table 2 would give a total passive pressure at E of 456kPa, which is a factor 1.33 greater than before. In order to get the same factor increase in total stress on the retained side, the effective earth pressure coefficient must rise to about 0.62. So the greatest credible increase in maximum bending moment, compared with the initial design value calculated earlier, is by a factor of about 1.33. This extremity represents an active mobilisation of only about 10° of internal friction on the retained side and a full passive mobilisation of 30° on the excavated side of the wall.

BS8002 recommends that, in general, the design bending resistance should equal the design bending moment calculated from the design earth pressures and design soil strengths based on a uniform mobilisation factor as expounded earlier. It does recognise, however, that conditions of swelling of clays beneath excavations could lead to the increased mobilisation of passive pressure. This is one of the contingencies, listed in section 3.1.9 of the Code, which could cause the designer to make a modest enhancement of the design bending moment, in the fashion explained above. The other contingencies are : high initial earth pressure, coupled with a construction method which permits very little stress relief; distortion of the whole retaining wall system due to subsidence arising from below; and heavy compaction against a rigidly propped face which later distorts when the props are removed. The generic syndrome is one in which the usual pattern of wall and soil movements during construction cannot occur. We have just demonstrated that, when an allowance for extra passive pressure should be made, the increase in bending moments might be only about 30%.

Conclusion

BS8002 marks a seachange in Codes of Practice. Compared with CP2 it enlists the aid of certain strong principles which reduce uncertainty in design.

(1) *Equilibrium.* It insists that an equilibrium free-body diagram be produced for every retaining structure. This is very helpful, especially, in determining the possible ranges of earth pressures to be used in design.

(2) *Effective stress analysis.* It insists that all materials in the ground obey Terzaghi's principle of effective stress and recognises that clays have an angle of friction. This provides a rationale for the old rule that clay be considered to act on a retaining wall with a pressure equivalent to a fluid with a 'density of 30 lbf/ft^3'.

(3) *Mobilisation.* It recognises that the peak strength of soil cannot generally be mobilised without excessive strain and proposes strength reduction (mobilisation) factors which aim to limit the lateral deflection of walls to about 0.5% of their height.

The new Code is open to scientific scrutiny, where previous Codes have made *ex cathedra* pronouncements. It is a limit state design Code based logically on the specification of a permissible, mobilisable, soil strength. It avoids the jungle of partial factors and looks to a future in which structural engineers request appropriate data from their site investigations, and then make appropriate use of them in design.

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